5.2. Let *L* and *M* be observables and  a normalized simultaneous eigenvector of *L* and *M*. Set (i)  and .

1. Show that 
2. Show that 
3. Show that (5.13) 

Solution. Recall that if  is the set of eigenvalues of an observable *L* then

(ii) 

where  is the probability that ** is the outcome of measurement *L*.

Notation: In the book, equation (4.11) used  because *L* was obvious. However, in this problem we have . For the sake of clarity, we write  rather than .

It is straight-forward to check that the eigenvalues and eigenvectors of  are , respectively. Since ,

(iii) .

1. Let  be the sets of eigenvalues of *L* and *M*, respectively. Define

(iv)  and  .

Claim  are the eigenvalues and eigenvectors of  and :



and similarly for *M*.

By (i), the distribution of  is simply the distribution of *L* shifted by  to have a mean of zero. Therefore, since  are the eigenvalues of ,

(v) ,

(vi) 

and similarly

(vii) .

Hence

 ✔

and similarly for *M*. ✔







Subtracting, we get

(viii)  ✔

1.  ✔

(\*) The book showed that this inequality, 5.13, holds because  = 0 = .

Note. I did not use step 1) in the proof of steps 2) and 3). Hence I also did not use (ii) – (v).